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Strain Rate Dependency on Stress–Strain Relations of Polypropylene

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Synopsis

True constant strain rate tensile tests of polypropylene up to 2% initial strain were performed to investigate the strain rate dependency of the initial stress–strain gradient $d\sigma/d\epsilon$ with the aid of "slack grip." The experimental results fairly agree with even the simplest mechanical model analysis with three parameters, showing the conspicuous strain rate dependency on the initial stress–strain gradient $d\sigma/d\epsilon$. The limiting cases corresponding to zero strain rate and strain rate of infinite magnitude, both predicted by the analysis, were experimentally verified.

INTRODUCTION

It is well known that linear polymers such as polypropylene show a conspicuous strain rate dependency of the stress–strain curve subjected to tension, increasing the initial gradient $d\sigma/d\epsilon$ with increase in strain rate $d\epsilon/dt = \dot{\epsilon}$.[1] In addition, it is not unreasonable to adopt a mechanical model to approximate the behavior of linear polymer for small amounts of initial strains.

In case a three-parameter model is adopted, it may be possible to represent this strain rate dependency analytically, and this strain rate dependency is further expected to have both upper and lower limiting bounds through the analysis.

On the other hand, the straight forward representation of results of so-called constant strain rate tensile tests, in other words, the constant cross-head speed tests, using commercial tensile testers, might not always indicate the true constant strain rate data in the strict sense, since every commercial tensile tester inherently has several nonconstant transient strain rate ranges, i.e., transient cross-head speeds of lower value, before reaching its prescribed constant cross-head speed, especially at high cross-head speed.

In the present investigation the true constant strain rate tensile tests up to 2% initial strain were carried out with the aid of a specially invented "slack grip" to obtain true constant strain rates for high strain rate tests. The analytical approach with the concept of a simplest three-parameter model is made to approximate the experimental results to give an interpretation for the strain rate dependent stress–strain gradient during the initial small strains. The upper and lower bounds expected in the analysis are also checked experimentally.

ANALYTICAL

Analytical Approach

The three-parameter mechanical model shown in Figure 1 which may be the simplest model for representing the existing material behavior, is taken to simulate the mechanical behavior of polypropylene. The corresponding stress–strain relations with the application of stress σ are

$$\sigma = E_1\epsilon_1 \tag{1}$$

for the Hookean model, and

$$\sigma = E_2\epsilon_2 + \eta\dot{\epsilon}_2 \tag{2}$$

for the Voigt model, where E_1 and E_2 are the Young's moduli, η is the viscosity coefficient, and ϵ_1 and ϵ_2 are the corresponding strains.

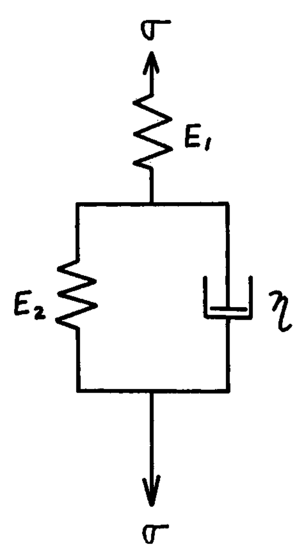

Fig. 1. Three-parameter mechanical model.

Introducing the Laplace transformation, we have

$$\bar{\sigma} = E_1\bar{\epsilon}_1 \tag{3}$$

$$\bar{\sigma} = (E_2 + S\eta)\bar{\epsilon}_2 \tag{4}$$

where S denotes a parameter appearing in the transformation.

Combination of eqs. (3) and (4) leads to

$$\bar{\sigma}(E_2 + S\eta) + E_1\bar{\sigma} = E_1(E_2 + S\eta)\bar{\epsilon} \tag{5}$$

where $\bar{\epsilon} = \bar{\epsilon}_1 + \bar{\epsilon}_2$, showing an image function of total strain ϵ.

Subsequent Laplace inverse transformation, after rearrangement, leads to

$$\sigma + \frac{\eta}{E_1 + E_2}\dot{\sigma} = \frac{E_1E_2}{E_1 + E_2}\epsilon + \frac{E_1\eta}{E_1 + E_2}\dot{\epsilon} \tag{6}$$

where $\dot{\sigma} = d\sigma/dt$.

Let $\dot{\epsilon}$ be constant, putting $\dot{\epsilon} = K$, then eq. (6) becomes

$$\dot{\sigma} + \frac{E_1 + E_2}{\eta}\sigma = \frac{E_1E_2}{\eta}\epsilon + E_1K \tag{7}$$

and finally the general solution for eq. (7) is

$$\sigma = -\frac{E_1{}^2\eta\, K}{(E_1 + E_2)^2}\mathrm{e}^{-(E_1+E_2/\eta)t} + \frac{E_1E_2}{E_1 + E_2}Kt + \frac{E_1{}^2\eta\, K}{(E_1 + E_2)^2} \tag{8}$$

with the initial condition of $\sigma = 0$ at $t = 0$.

Substituting $t = \epsilon/K$ into eq. (8), we obtain

$$\sigma = \frac{E_1{}^2\eta K}{(E_1 + E_2)^2}(1 - \mathrm{e}^{-(E_1+E_2)\epsilon/(\eta K)}) + \frac{E_1E_2}{E_1 + E_2}\epsilon \tag{9}$$

in which K shows the strain rate contribution to the stress–strain relation.

As extreme cases, both $K = 0$ and $K = \infty$ are investigated. First, for $K = 0$, eq. (9) is reduced to

$$\sigma = \frac{E_1E_2}{E_1 + E_2}\epsilon \tag{10}$$

and for $K = \infty$, with the expansion in series concerning the exponential term, eq. (9) leads to

$$\sigma = E_1\epsilon \tag{11}$$

Therefore the stress–strain curve obtained at any strain rate must be between those two limiting curves given by eqs. (10) and (11).

EXPERIMENTAL

Model Characterization of Polypropylene

Since polypropylene is a linear polymer, a four-parameter model, shown in Figure 2, may generally be applied. These four parameters can be decided by either the creep test or the stress relaxation test. In the present study, the creep test was done, whose creep curve is shown in Figure 3, and $E_1 = 96.6$ kg/mm², $E_2 = 76.0$ kg/mm², and $\eta_1 = 1.34 \times 10^6$ kg sec/mm² are determined for constant creep stress $\sigma_0 = 1.40$ kg/mm² at 22°C.

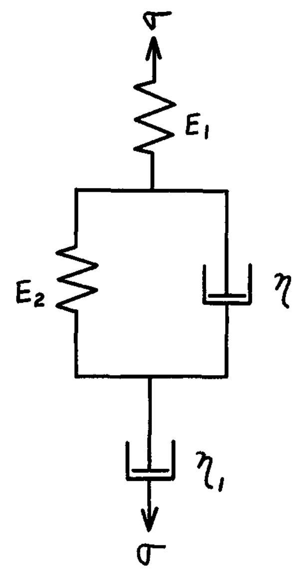

Fig. 2. Four-parameter mechanical model.

The η_2 value is determined at the inflection point in the creep strain-versus-time curve shown in Figure 4 in terms of $\eta_2 = E_2\tau$, where τ is the retardation time. Thus τ is decided to be 0.095 sec, hence $\eta_2 = 7.22$ kg sec/mm².

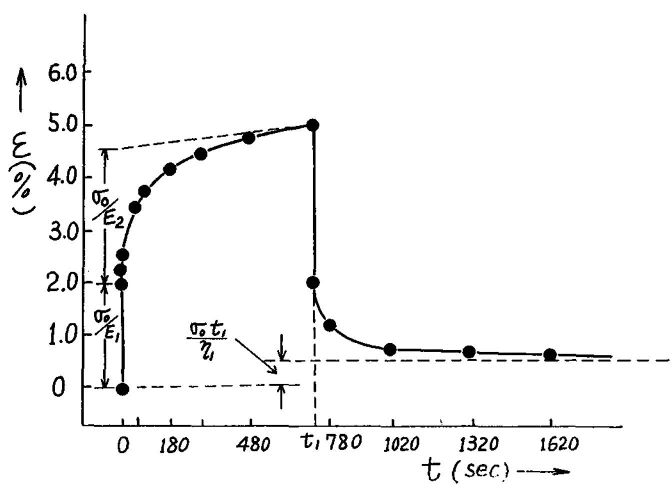

Fig. 3. Creep curve of polypropylene.

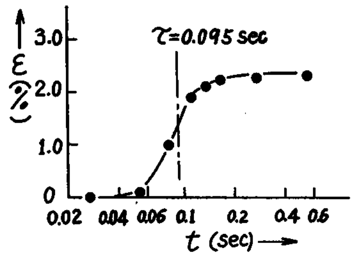

Fig. 4. Creep strain-vs.-time curve to determine retardation time of polypropylene.

Because η_1 is far greater than η_2, η_1 may be considered further as a viscous element, say, $\eta_1 = \infty$, for simplicity. Now, the subsequent mechanical model turns out to be a three-parameter model having $E_1 = 96.6$ kg/mm², $E_2 = 76.0$ kg/mm², and $\eta_2 = \eta = 7.22$ kg sec/mm².

Actually, since the initial stress–strain gradient for initial times is the subject of interest, and η_1, which becomes effective after long times have elapsed, makes no contribution, therefore, in the present paper, this three-parameter model is used as a mechanical equivalent of polypropylene.

Tensile Testers

The usual Instron-type tensile testers were used to give a constant strain rate tension ranging from 1.53×10^{-4}/sec to 156/sec to the specimen, as shown in Table I. Of course, as mentioned previously, every tensile tester has a nonconstant transient strain rate range before reaching the prescribed strain rate, especially at high cross-head speed, whose effects were eliminated in the present test by using the special "slack grip."

For $\dot{\epsilon}$ Ranging from 1.53×10^{-4}/sec to 1.48×10^{-2}/sec. The TOM-2000-type tensile tester was used; nominally it has no transient strain rate range before reaching the prescribed tensile cross-head speed. Actually, careful examination of elongation-versus-time data obtained by use of the ET/5 elongation meter, by which elongation is converted into electrical output through the strain gauge placed on the cantilever mechanism inside the elongation meter (shown in Figure 5) shows quite negligible nonconstant cross-head speed range. Therefore, no special "slack grip" is used.

For $\dot{\epsilon} = 4.16 \times 10^{-1}$/sec. Shimadzu's IM-100-type tensile tester was used. This tester has a time lag of 0.3 sec until it reaches $\dot{\epsilon} = 4.16 \times 10^{-1}$/sec, i.e., corresponding to 5 mm of transient strain rate range. Therefore, the specially designed "slack grip" to give a constant strain rate to a specimen was employed, as shown in Figure 6. Here, the test specimen is subjected to tension at constant cross-head speed after the connecting rod runs downward more than 120 mm in stroke. The 120 mm of idle stroke is determined so that the UTM-5-type tensile tester, mentioned below, may have enough margin in the constant cross-head speed range, since the UTM-5 tester has a transient cross-head speed range of 80 mm, as shown in Fig. 7.

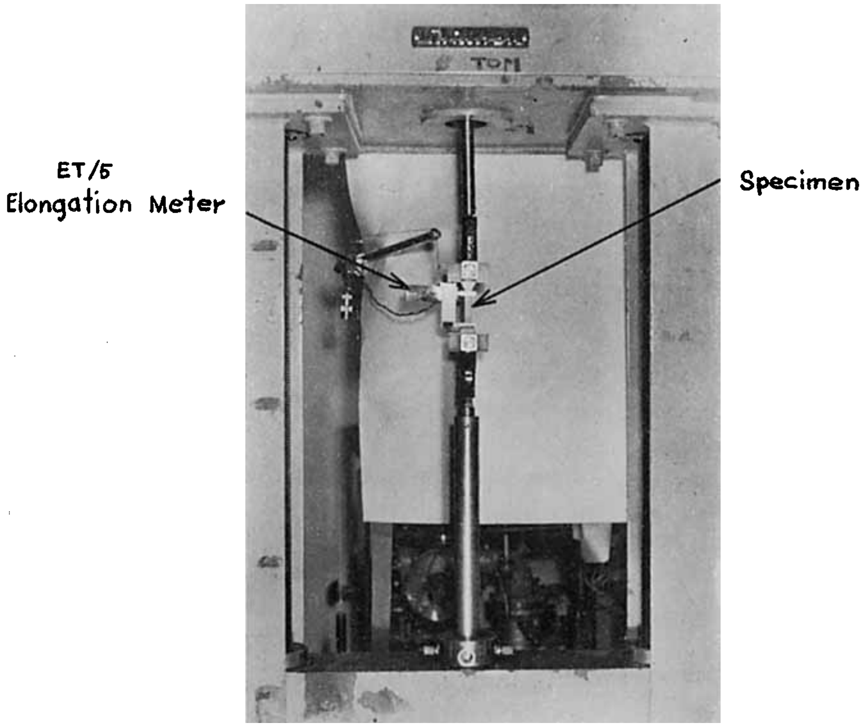

Fig. 5. TOM 2000 Tensile tester and ET/5 elongation meter.

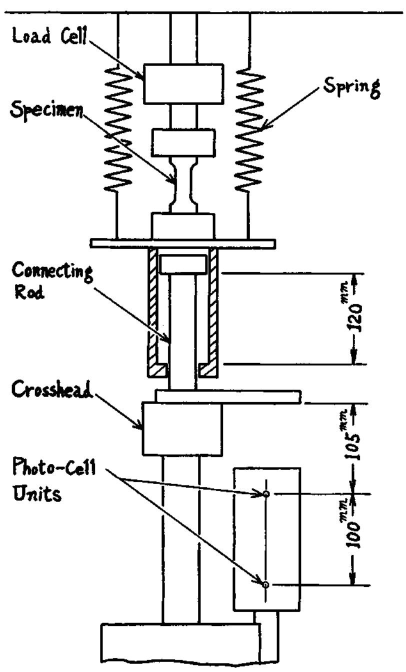

Fig. 6. Specially designed "slack grip" to achieve the true constant strain rate tension of a specimen.

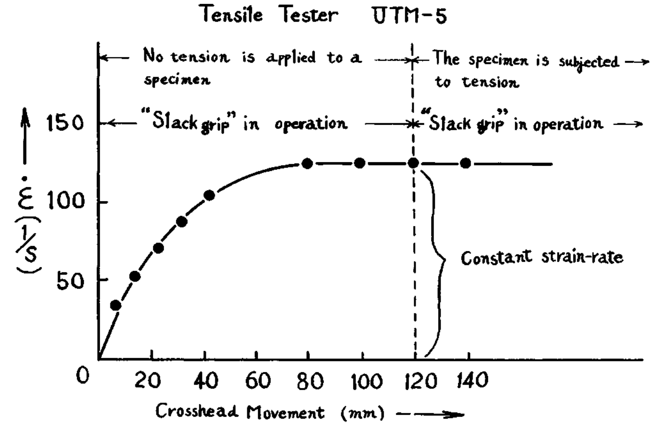

Fig. 7. Example of inherent transient strain rate range of UTM-5 tensile tester.

For $\dot{\epsilon}$ Ranging from 41.6/sec to 156/sec. Further, for higher strain rates, say, $\dot{\epsilon}$ = 41.6/sec, 103/sec, and 156/sec, the UTM-5-type tensile tester, specially designed for higher strain rate tension loading, mostly for elastomers, was used. The inherent transient strain rate range is shown in Figure 7. This transient cross-head speed range is inevitable, since the UTM-5-type tester is designed and manufactured to fit elastomers showing

TABLE I
Experimental Apparatus

$\dot{\epsilon}$, 1/sec	Tensile tester	Transient range	Load cell	Recorder
1.53×10^{-4} 6.22×10^{-4} 1.48×10^{-2}	TOM-2000 (Shinkoh Communication Industry Co., Ltd., Japan) (max. 5000 kg tension)	nominally zero[a]	conventional strain-gauge type, max. 100 kg (Kyowa Electronic Instruments Co., Ltd., Japan)	electromagnetic oscillograph EMO-1 (Yokogawa Electric Works Ltd., Japan)
4.16×10^{-1}	IM-100 (Shimadzu Seisakusho Ltd., Japan) (max. 100 kg tension)	5 mm[b]		
41.6 103 156	UTM-5 (Toyo Measuring Instruments Co., Ltd., Japan) (max. 500 kg tension)	80 mm[b]	semiconductor type, max. 500 kg (Toyo Measuring Instruments Co., Ltd., Japan)	Memoriscope[c] MS-5507 (Iwatsu Electric Co., Ltd., Japan)

[a] No special device such as "slack grip" was used, becuase of no transient cross-head speed.

[b] Special "slack grip" to obtain constant strain rate without transient strain rate range was invented to be attached.

[c] Stocked data were photographed by a Polaroid camera. Memoriscope is a data storage-type cathode-ray oscillograph tube.

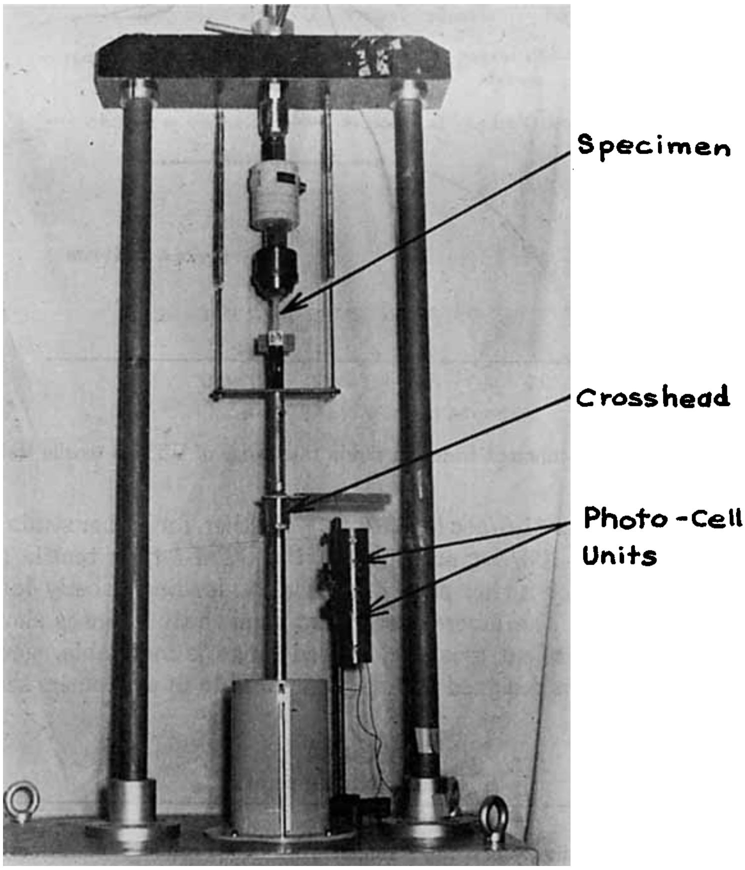

Fig. 8. UTM-5 tensile tester and photocell units.

large elongations. Therefore, the special "slack grip" mentioned above was fitted, and the tensile cross-head speed was surveyed by measuring the time elapsed to cross the distance between two photocell units, as shown in Figure 8.

Load Cells

The test load was measured by use of a load cell. Especially for higher strain rate tests, the semiconductor load cell specially designed for rapid elongation, with maximum capacity of 500 kg tension, was used (Table I).

Data Recording

For $\dot{\epsilon} = 1.53 \times 10^{-4}$/sec 6.22×10^{-4}/sec, 1.48×10^{-2}/sec, and 4.16×10^{-1}/sec, both elongation-versus-time curves and load-versus-time curves were recorded by the electromagnetic oscillograph type EMO-1. For higher strain rates of $\dot{\epsilon}$ = 41.6/sec, 103/sec and 156/sec, the load-versus-elongation curves were directly obtained and stocked on the Memoriscope

MS-5507 (Table I), and afterward these stocked data were photographed by a Polaroid camera.

Test Specimens

The test specimen, shown in Figure 9, was stamped out of a 2 mm-thick virgin polypropylene sheet, Chisso Polypro-1011, manufactured by Chisso Corporation, Japan.

The strain rate $\dot{\epsilon}$ used in the present study was (prescribed cross-head speed)/40 mm.

The test temperature was 22°C or 23°C.

Data Reduction

For Lower Strain Rates. As mentioned previously, for $\dot{\epsilon}$ ranging from 1.53×10^{-4}/sec to 1.48×10^{-2}/sec actually no transient cross-head speed range exists. Therefore, no special device such as a "slack grip" was necessary, and the straightforward representation of data was accepted.

For Higher Strain Rates. For $\dot{\epsilon} = 4.16 \times 10^{-1}$/sec to 156/sec, the special "slack grip" to achieve the constant strain rate tension was attached; therefore, no transient effects existed. However, another phenomenon (shown in Fig. 10) was observed, which is due to resonance of the entire load-measuring system using a load cell.[2] Data reduction in this case was applied so that true stress may be obtained. That is, the true stress was taken at the midpoint of every amplitude, of which an example is shown in Figure 11.[3]

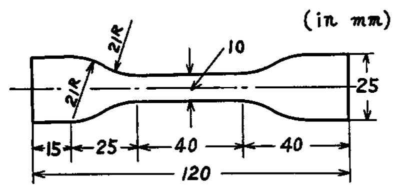

Fig. 9. Configuration and dimensions of test specimen.

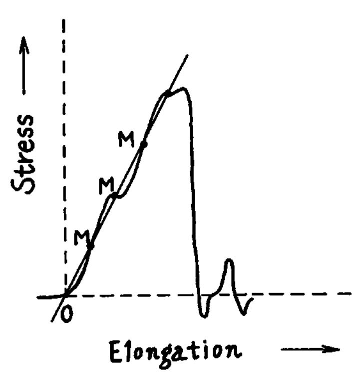

Fig. 10. Example of load-vs.-elongation curve obtained on the memoriscope (data storage type CRT).

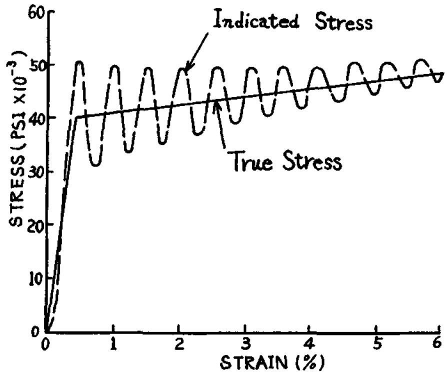

Fig. 11. True stress determined from fluctuating indicated stress.

Experimental Results and Discussion

The experimental results thus obtained are shown in Figure 12. The solid lines show the analytical curves calculated from eq. (9), and upper, average, and lower values are given for each individual experiment.

The stress-versus-strain data (σ versus ϵ) for polypropylene during initial small strains at various constant true strain rates were obtained as shown in Figure 12, showing the distinct strain rate dependency of the initial stress–strain gradient $d\sigma/d\epsilon$.

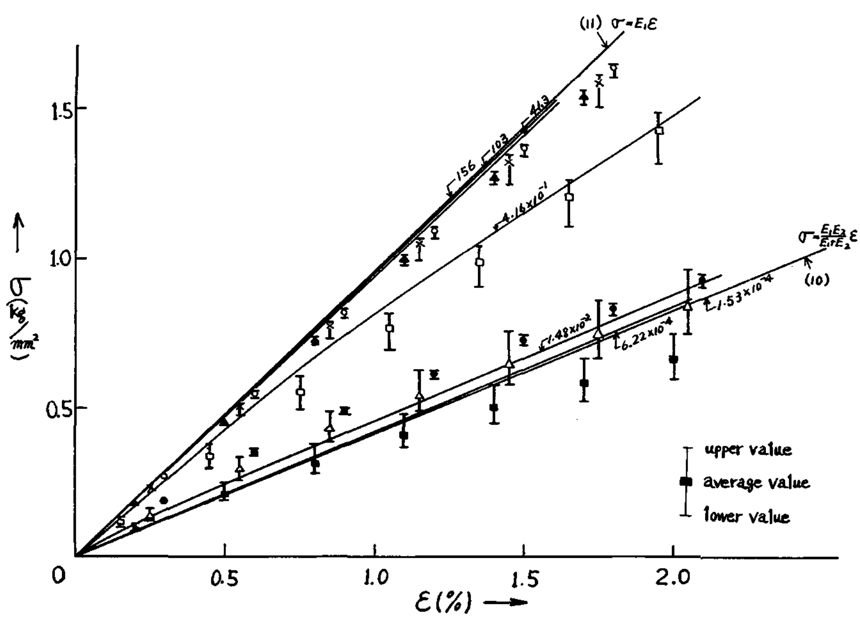

Fig. 12. Experimental results in comparison with analytical curves: (■) $\dot{\epsilon}$ = 1.53 × 10^{-4}/sec; (△) $\dot{\epsilon}$ = 6.22 × 10^{-4}/sec; (●) $\dot{\epsilon}$ = 1.48 × 10^{-2}/sec; (□) $\dot{\epsilon}$ = 4.16 × 10^{-1}/sec; (▲) $\dot{\epsilon}$ = 41.3/sec; (×) $\dot{\epsilon}$ = 103/sec; (O) $\dot{\epsilon}$ = 156/sec.

When the analytical approach is made through a simple three-parameter model, the general tendency is agreement between the analytical and the experimental results, in spite of the simple mechanical model. Of course, in Figure 12 discrepancies are seen in the details, since the original assumption of constant E_1, E_2, and η of the three-parameter model, obtained from the creep test, prevents these constants from any variation due to change in applied stress and the corresponding strain. For higher strain rates, say, 4.16×10^{-1}/sec, 41.3/sec, 103/sec, and 156/sec, the comparison between the analytical and the experimental results shows rather close agreement as a rule. However, for lower strain rates, say, 1.53×10^{-4}/sec, 6.22×10^{-4}/sec, and 1.48×10^{-2}/sec, discrepancies between the analytical curves and the experimental data are observed, because in the analysis a dashpot η_1, which is more effective at lower strain rates, is regarded as infinity since $\eta = \eta_2 \ll \eta_1$.

As shown in the analytical approach, so far as the three-parameter model is concerned, there exist two limiting bounds, i.e., the upper bound corresponding to an infinite strain rate shown in eq. (11), and the lower bound corresponding to a zero strain rate shown in eq. (10). Figure 12 shows that the experimental values of higher strain rates, i.e., 41.3/sec, 103/sec, and 156/sec, seem to approach the upper bound asymptotically; however, those of the lower strain rate of 1.53×10^{-4}/sec show rather a large discrepancy with the lower limit for the strain range larger than 0.8%. This discrepancy in the lower strain rate is probably due to the substitution of η_1 dashpot with that of infinite viscosity, as mentioned above.

CONCLUSIONS

True strain rate tensile tests of polypropylene up to 2% initial strain were performed to investigate the strain rate dependency of the initial stress-strain gradient $d\sigma/d\epsilon$. Fairly good agreement was seen between the experimental data and analytical results, even with the simplest mechanical model, a three-parameter model showing a conspicuous strain rate dependent stress-strain relation. The existence of upper and lower limiting values predicted by the analytical approach were also verified experimentally.

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